2. V. A. Nikolaeva, V. I. Ryzhii, and B. N. Chetverushkin, "Numerical modeling of a dense electron—hole plasma in binary heterostructures," Preprint No. 38, IPM, Akad. Nauk. SSSR, Moscow (1985).
3. B. S. Kerner and V. V. Osipov, Zh. Eksp. Teor. Fiz., 71, 1542-1554 (1976).
4. B. S. Kerner and V. V. Osipov, Fiz. Tekh. Poluprovodn., 13, 721-734 (1979).
5. B. A. Nikolaeva, V. I. Ryzhii, and B. I. Chetverushkin, Inzh.-Fiz. Zh., 51, No. 3, 492-501 (1986).

STABILITY AND FORM OF AN ARC COLUMN IN A TRANSVERSE GAS FLOW

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The analytic conditions for stable burning of an electric arc in a gas flow are obtained. The form of the arc column in a transverse gas flow is determined.

A great deal of attention is being devoted to the problem of the stability of an electric arc. A number of authors [1-3] have studied the spatial stability of an arc neglecting the effect of the electric circuit by introducing the concepts of mass and nonmass forces. Others have studied the energy stability of electric arc circuits without a gas flow [4-6] and with a gas flow [7-8], neglecting the details of the motion of the arc in space. The spatial and energy stability of different electric arcs is studied simultaneously in [9] using the "force" model. The results on the stability are based on an analysis of the forces which act on the arc and which are difficult to determine in experiments.

The form of the arc column has been studied quite completely for an arc in a longitudinal flow. As regards an arc in a transverse flow, the form of the arc is determined primarily experimentally.

We shall study a cylindrical arc stabilized by electrodes in a transverse gas flow. We shall assume that there is no external magnetic field, while the intrinsic magnetic field is negligibly small. Then the equation expressing the balance of energy in the arc for a linear dependence of the gas properties on the enthalpy has the form [10]

$$\frac{\partial \overline{h}}{\partial t} + (\mathbf{w},\ \nabla \overline{h}) = a^2 \nabla^2 \overline{h} + (\varepsilon^2 - \overline{\varepsilon^2})\, \overline{h}, \tag{1}$$

where $\overline{\varepsilon^2}$ takes into account the radiative heat losses. We assume that the intensity of the electric field in the arc ε depends on t and varies only along the arc, since, as investigations have shown [5, 6], the arc is stable with respect to the transverse perturbations of the field.

Averaging Eq. (1) over the period of oscillations of the current in the circuit, if the arc is an ac current arc, and integrating over the cross section of the arc column, in the presence of perturbations in the system we obtain

$$\frac{\partial H}{\partial t} = a^2 \frac{\partial^2 H}{\partial x^2} + p - q + F(x,\ t), \tag{2}$$

where p and q are, respectively, the Joule heat and heat losses to the environment per unit arc length; F(x, t) is the external perturbation. The overbar indicating averaging over the period is dropped here and below.

Equation (2) must be supplemented by Ohm's law for the circuit and relations for p and q:

$$\int_{-l/2}^{l/2} \varepsilon(x,\ t) \sqrt{1 + \left(\frac{\partial y}{\partial x}\right)^2}\, dx = l \left(\varepsilon_s - \frac{RI}{l} \sqrt{\frac{\overline{\sigma}}{\overline{M}}}\right), \tag{3}$$

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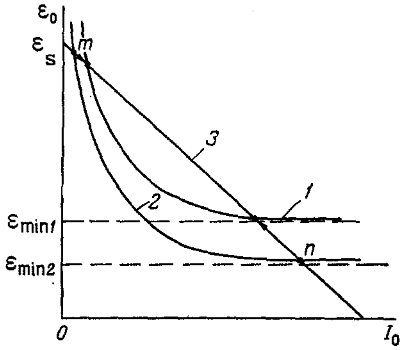

Fig. 1. Displacement of the working points with an increase in the velocity of the gas flow v_0: 1) IVC (v_0*); 2) IVC (v_0**); 3) load line; v_0* > v_0**.

$$p = \frac{I\varepsilon}{\sqrt{\bar{\sigma}\bar{M}}}. \tag{4}$$

We do not write out the expression for q, since for what follows only the equality $p_0 = q_0$ in the equilibrium state will be important. The coordinate system x, y is chosen so that the x axis is oriented perpendicularly to the gas flow, the y axis coincides with the direction of the flow, and the origin of coordinates is located midway between the electrodes.

We shall study the stability of the discharge with respect to small and smooth perturbations in the circuit or in the gas flow. It is clear that in the case of the stable state the response of the arc to such perturbations will also be small and smooth. This implies that the reactive elements of the circuit can be neglected. We note that the reaction of the arc to external perturbations cannot reduce merely to changes in H, I, and ε. Because of the fact that the working point of the current–voltage characteristic (IVC) of the arc can move only along the load line of the circuit, while the position of the IVC, according to our model, depends only on the flow velocity of the gas v_0, any perturbation of the arc to move. This is confirmed by numerous experimental studies [1, 9]. For this reason, in what follows, we assume that the quantities H, ε, p, and q depend on the local relative velocity of the flow past the arc $v(x, t) = v_0(t) - v_a(x, t)$. Here $v_0(t)$ is in general, a slowly varying function of time, while $v_a(x, t)$ is the velocity of a fixed point in the arc. Assuming that $|v_a(x, t)| \ll v_0(t)$, we expand all quantities in a series in v_a up to linear terms, and we represent the current strength in the form $I(t) = I_0(v_0) + I_1(t)$, since it is independent of the coordinates. Then we obtain the following expressions for the Joule heating p and the derivative q':

$$p(x, t) = p_0(v_0) + \frac{1}{\sqrt{\bar{\sigma}\bar{M}}}(\varepsilon_0 I_1 - I_0 \varepsilon_0' v_a), \tag{5}$$

$$q_0' = \frac{\varepsilon_0'}{\sqrt{\bar{\sigma}\bar{M}}}\left(I_0 + \sqrt{\frac{\bar{M}}{\bar{\sigma}}}\frac{\varepsilon_0 l_a}{R_a}\right), \tag{6}$$

where R_a is the differential resistance of an equilibrium arc discharge; $\ell_a = \int_{-l/2}^{l/2}\sqrt{1 + \left(\frac{\partial y}{\partial x}\right)^2}\,dx \approx \ell$ up to second-order infinitesimals. Using the expansion of H in powers of v_a, taking into account (5) and (6), from (2) and (3) we find

$$\frac{\partial v_a}{\partial t} = a^2\frac{\partial^2 v_a}{\partial x^2} - \frac{1}{\sqrt{\bar{\sigma}\bar{M}}}\frac{\varepsilon_0}{H_0'}\left(I_1 + \frac{l}{R_a}\sqrt{\frac{\bar{M}}{\bar{\sigma}}}\varepsilon_0' v_a\right) - \frac{F(x, t)}{H_0'} + \dot{v}_0, \tag{7}$$

$$\int_{-l/2}^{l/2}\left[\varepsilon_0' v_a - \frac{\varepsilon_0}{2}\left(\frac{\partial y}{\partial x}\right)^2\right]dx = \sqrt{\frac{\bar{\sigma}}{\bar{M}}}RI_1. \tag{8}$$

Dropping in (8) the term $(\partial y/\partial x)^2$, as a second-order infinitesimal, after integrating (7) over dx we obtain

$$\frac{dI_1}{dt}=\sqrt{\frac{\overline{M}}{\overline{\sigma}}}a^2\frac{\varepsilon_0'}{R}\left(\frac{\partial v_a(l/2,\ t)}{\partial x}-\frac{\partial v_a(-l/2,\ t)}{\partial x}\right)-$$
$$-\frac{l\varepsilon_0}{\overline{\sigma}}\frac{\varepsilon_0'}{H_0'}\left(\frac{1}{R}+\frac{1}{R_a}\right)I_1-\sqrt{\frac{\overline{M}}{\overline{\sigma}}}\frac{1}{R}\frac{\varepsilon_0'}{H_0'}Q(t)+\sqrt{\frac{\overline{M}}{\overline{\sigma}}}\frac{l\varepsilon_0'}{R}\dot{v}_0(t). \tag{9}$$

Here $Q(t) = \int_{-l/2}^{l/2} F(x,\ t)\,dx$. We note that the first term on the right side of (9) is proportional to the perturbation of the heat flow in the electrodes, which in its turn is proportional to U_eI_1, where U_e is the sum of the voltage drops near the electrodes.

From the relations $I_0 = \sqrt{\overline{\sigma}\overline{M}}\, H_0\varepsilon_0/\alpha$ and $\varepsilon_0 = \varepsilon_s - \sqrt{\overline{\sigma}/\overline{M}}\, RI_0/\ell$ (see [10] for a discussion of α) it is easy to obtain

$$\frac{\varepsilon_0'}{H_0'}=-\frac{\overline{\sigma}\varepsilon_0}{\dfrac{l\alpha}{R}+\overline{\sigma}H_0}.$$

Depending on the choice of working point on the IVC of the arc ε'_0 will either be negative (the point m in the figure) or positive (the point n). The same is true for the sum $(R^{-1} + R_a^{-1})$. Therefore (9) will have a different form for the working points m and n:

$$\frac{dI_1}{dt}=\pm AI_1\pm BI_1+CQ(t)\mp D\dot{v}_0. \tag{10}$$

Here the upper (lower) sign corresponds to the choice of working point m(n), and the letters A, B, C, and D denote the coefficients in front of the corresponding terms on the right side of (9). According to (10), I_1 increases exponentially at the working point m which agrees with the well-known result of [4, 7, 8] regarding the instability of the circuit with an arc at this point. Conversely, the solution of (10) at the point n is stable.

We return to the equation (7). Its solution is stable only at the point n. Let us examine the asymptotic solution of (7) for the case when the perturbations $v_0(t)$ and $F(x, t)$ act for a finite time τ_1. Then, integrating (7) over dt from zero to $\tau \gg \tau_1$ and taking into account the fact that $v_a(\tau) = 0$ and $v_a = \partial y/\partial t$, we find

$$a^2\frac{d^2y}{dx^2}-s^2y+\Delta v_0+\overline{Q}(x)+k^2\overline{q}=0, \tag{11}$$

where

$$s^2=\frac{\varepsilon_0'}{H_0'}\frac{\varepsilon_0 i}{R_a\overline{\sigma}};\quad k^2=-\frac{1}{\sqrt{\overline{\sigma}\overline{M}}}\frac{\varepsilon_0}{H_0'};$$
$$\overline{Q}(x)=-\frac{1}{H_0'}\int_0^\tau F(x,\ t)\,dt;\quad \overline{q}=\int_0^\tau I_1(t)\,dt;\quad \Delta v_0=\int_0^\tau \dot{v}_0dt.$$

The solution of (11) for the case when $\bar{Q}$ is independent of x (the perturbation is uniform over the length of the arc) has the form

$$y=\frac{\Delta v_0+\overline{Q}+k^2\overline{q}}{s^2}\left(1-\frac{\operatorname{ch}\dfrac{s}{a}x}{\operatorname{ch}\dfrac{sl}{2a}}\right).$$

As we can see, the arc has the form of a catenary, which agrees with the observations of many investigators. Since $y \sim 1/s^2$, while $s \sim I_0{}^2$, as the current in the arc increases the arc becomes increasingly less sensitive to the external perturbations, i.e., the arc becomes more "inertial."

NOTATION

h, enthalpy; t, time; w, mass velocity; a^2, coefficient of thermal diffusivity; ε, intensity of the electric field; ℓ, distance between the electrodes; H, enthalpy per unit length of the arc column, averaged over the period; I, current strength; $\varepsilon_s\ell$, voltage of the current source; $\bar{\sigma}$ and $\bar{M}$, coefficients in the linear approximation for the electrical conductivity and density; R, active resistance of the chain; v, velocity. Indices: 0, equilibrium state of the arc; a, arc; ℓ, differentiation with respect to the velocity v_0; an overdot indicates a time derivative.

LITERATURE CITED

1. G. Mekker, TIIÉR, 59, No. 4, 4-14 (1971).
2. D. E. Anderson, Transport Phenomena in Thermal Plasma [Russian translation], Moscow (1972).
3. P. A. Kulakov, Thermophysics and Physical Hydrodynamics [in Russian], Novosibirsk (1978), pp. 126-134.
4. A. A. Andronov, A. A. Vitt, and S. É. Khaikin, Theory of Oscillations [in Russian], Moscow (1981).
5. V. D. Khait, Teplofiz. Vys. Temp., 18, No. 3, 489-496 (1980).
6. V. I. Artemov, I. N. Rutkevich, and O. A. Sinkevich, Teplofiz. Vys. Temp., 18, No. 6, 1126-1136 (1980).
7. M. F. Zhukov, A. S. Koroteev, and B. A. Uryukov, Applied Dynamics of Thermal Plasma [in Russian], Novosibirsk (1975).
8. M. F. Zhukov (ed.), Theory of Electric Arc Under Conditions of Forced Heat Exchange [in Russian], Novosibirsk (1977).
9. O. Ya. Novikov, Stability of an Electric Arc [in Russian], Leningrad (1978).
10. N. V. Pashatskii and S. G. Lisitsyn, Inzh.-Fiz. Zh., 47, 133-139 (1984).

COMPUTATION OF THE DYNAMICS OF SWIRLING VORTEX FLOW IN A CYLINDER OF AN INTERNAL-COMBUSTION ENGINE DURING THE COMPRESSION STROKE

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The vortex flow configuration in an engine cylinder is calculated for various geometries of an undivided combustion chamber. The data of a numerical analysis and experimental measurements are compared.

1. Three types of internal-combustion (IC) engine models are known in the literature [1-3]. The first type refers to thermodynamic or zero-dimensional models [4], whose parameters (velocity, temperature, concentration, pressure, etc.) are averaged over the volume of the combustion chamber and are functions of only one variable: time [5-8]. The analyzed region of the combustion chamber is partitioned into the zones of the fresh charge and the combustion products; the partition of each of them is described by thermodynamic relations formulated on the basis of the ideal-gas energy balance equations and equations of state. Thermodynamic models are concerned primarily with the analysis of turbulent mixing rates and the influence of turbulence on the flame propagation velocity. Combustion is treated here as the propagation of a laminar flame front of finite width via discrete turbulent vortices. An elaboration of this approach may be found in [8], where the mixing and combustion processes are described (with allowance for semiempirical information on the individual structure of the vortices) by statistical modeling.

In quasidimensional models [9, 10], the machinery developed in thermodynamic models is augmented with a semiempirical theory of the propagation of a one-dimensional turbulent flame.

The simplifications associated with the formulation of zero-dimensional and quasidimensional models are significantly manifested in the modeling of stratified-charge fuel-injection engines and in diesel engines, where the rate of heat release through chemical reactions is primarily mixing-controlled. Detailed engine models have been proposed in a number of papers [11-19] in order to describe the mixing processes with allowance for the contri-

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